THE OUTLINE METHOD IN MATHEMATICS.

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Printed outlines have been used in many school subjects for some time, and, I think, have in general been found helpful. Moreover, by excluding the non-essentials of a subject, these outlines can hold up for closer inspection and emphasis the essentials of that subject. This is surely a strong point, and one that the large textbooks cannot claim. Work of this sort has been carried on in English, history, geography, civics, etc., but the secondary mathematical field has been comparatively neglected. I have for some years been using, in manuscript or printed form, outlines in algebra and geometry. These outlines are made the basis of the recitation and the assignment. The original textbook then becomes a reference and supply book.

Sample pages of algebra and geometry follow:

FACTORING.

Factoring is the reverse of Special Products. There are four common cases.

1. ANY NUMBER OF TERMS from which a common factor can be divided.
Formula: See case 1, special products.

2. TWO TERMS in the form of the difference of two squares.
Formula: See case 2, special products.

3. THREE TERMS in the form of the quadratic trinomial.
Formula: See case 5, specal products.

4. TWO TERMS in the form of the sum or difference of two like odd powers.
Formulas:

These four cases cover all the common methods of factoring. In any case the terms can be simple or parenthetical.

Case 2 can have three, four, or six terms, but they must then be grouped into two terms.

Case 3 can have four, five, or six terms, but they must then be grouped into three terms.

In case 4, even exponents can be considered odd except powers of 2. Thus:

ANOTHER METHOD OF FACTORING IS THE FACTOR THEOREM: If a polynomial in X equals zero when A is substituted for X, then X—A is a factor of the polynomial.

THE FOUR STEPS IN SYSTEMATIC FACTORING ARE:

1. Try case 1.
2. If there are only two terms, try case 2, or 4.
3. If there are only three terms, try case 3.
4. Try grouping into these cases, or else the factor theorem.

QUESTIONS ON FACTORING.

1. Algebraic factoring is the reverse of what?
2. State the four common cases of factoring.
3. State the formula of each.
4. Explain how polynomials of more than two terms can sometimes be factored by the second case.
5. Explain how polynomials of more than three terms can sometimes be factored by the third case.
6. What even exponents can be considered odd, if desired?
7. State the factor theorem.
8. State the four steps in systematic factoring.
9. If both case two and case four can be used, which is to be preferred?

CHAPTER XIX.

PROPORTION FROM SIMILAR TRIANGLES.

THE CORRESPONDING SIDES OF SIMILAR TRIANGLES ARE IN PROPORTION. (259)

269. *The corresponding altitudes of two similar triangles are in proportion with any two corresponding sides.*

271. *If two chords intersect within a circle, the product of the parts of one chord equals the product of the parts of the other chord.*

272. *If two secants intersect without a circle, the product of one whole secant and its external part equals the product of the other whole secant and its external part.*

273. *If a secant and a tangent intersect without a circle, the product of the whole secant and its external part equals the square of the tangent.*

276. *If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse:*

   1. *The triangles formed are similar to the given triangle and to each other.*
   2. *The perpendicular is the mean proportional between the parts of the hypotenuse.*
   3. *Either arm is the mean proportional between the whole hypotenuse and its adjacent part.*

277. *The perpendicular from any point in the circumference of a circle to the diameter is a mean proportional between the parts of the diameter.*

PROBLEM: Construct a mean proportional between two given straight lines.

SUMMARY.

279. State two new methods of proving lines in proportion. Recall the other two methods.

280. State three methods of proving a line a mean proportional between two other straight lines.

DISCUSSION (OF CHAP. XIX).

The proof of lines in proportion from similar triangle has two parts: (1) Proving triangles similar by one of the summary, Art. 267; (2) The corresponding sides of similar triangles are in proportion.

The proof that the product of two lines equals the product of two other lines has three parts: (1) Proving triangles similar; (2) Corresponding sides in proportion; (3) Product of the means equals product of the extremes, Art. 241.

269. Method is 259. The same thing can be proved of medians.

271. Method is 259 and 241. Join the ends of two chords.

272. Method is 259 and 241. Join the alternate intersections.

273. Method is 259 and 241. Join the intersection points.

274. The last three theorems can be grouped under one statement. If two chords intersect (internally or externally), the product of the parts of one chord equals the product of the parts of the other.

276. Method is: (1) Art. 265; (2) Art. 259; (3) Art. 259.

277. Method is 276, 2.

278. Method is 277.

279. The four methods are 244, 249, 259, and 274.

280. The three methods are 273; 276, 2; 276, 3.

The teacher introduces a topic by discussing its nature and methods as in the outline. Comparison, contrast, etc., should be used in relating the new to the old. The pupil then makes a model under the teacher's guidance and inserts it in the outline. This is followed by exercises of this type.

In algebra there is usually room on the pages for the models. In geometry any proof, if desired, is written on plain paper and inserted. There are eight theorems in geometry whose proofs are not always interchangeable in the common textbooks. Each of these has references to books where a proof suited to the order of the outlines can be found.

Results from the use of these outlines seem to show that the pupil is usually interested in making his notebook. It is his own

handiwork, and he will often be found working on this when he should be doing something else. Second, if he has difficulty at any time with his exercises, he knows where to look for his model, which he has made and which he is, therefore more likely to understand. Third, he gets a clear view of a topic as a whole, and of its methods. He cannot emphasize non-essentials, because they are not in sight. Fourth, there is sound training in habits of systematic effort. He is taught to classify the work he is about to do, and then recall the methods of that type.

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